Published by Institute of Physics Publishing for SISSA/ISAS
Received: *November 26, 2004*
Accepted: *February 16, 2005*

Non-minimally coupled tachyonic inflation in warped string background

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Abstract: We show that the non-minimal coupling of tachyon field to the scalar curvature, as proposed by Piao et al, with the chosen coupling parameter does not produce the effective potential where the tachyon field can roll down from $T = 0$ to large T along the slope of the potential. We find a correct choice of the parameters which ensures this requirement and support slow-roll inflation. However, we find that the cosmological parameter found from the analysis of the theory are not in the range obtained from observations. We then invoke warped compactification and varying dilaton field over the compact manifold, as proposed by Raeymaekers, to show that in such a setup the observed parameter space can be ensured.

Keywords: Tachyon Condensation, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Inflation [1] is possibly the only known mechanism which dynamically solves the flatness and the horizon problem of the universe. Thus it has become an almost indispensible ingredient in cosmology. The inflaton, a scalar field, can also produce the density perturbations causally which can match with the data from observation. For example, the recent WMAP data [2] strongly supports the idea that the early universe went through an inflationary phase. Usually one considers the inflationary phase to be driven by the potential of a scalar field. Recently there has been an upsurge in activity for constructing such models in string theory, for example see [3] for a review. In the context of string theory, the tachyon field in the world volume theory of the open string stretched between a D-brane and an anti-D-brane or on a non-BPS D-brane has been taken as a natural candidate to play the role of the inflaton [4]. This possibility of the tachyon field driving the cosmological inflation is related to the decay of unstable brane as a time dependent process which was advocated by Sen [5]. The effective action used in the study of tachyon cosmology consists of the standard Einstein-Hilbert action and an effective action for the tachyon field on unstable D-brane or brane-antibrane system. What distinguishes the tachyon action from the standard Klein-Gordon form for scalar field is that the tachyon action, as we will see in the next section, is non-standard and is of the Dirac-Born-Infeld form [6]. The tachyon potential is derived from string theory itself and has to satisfy some definite properties to describe tachyon condensation and other requirements in string theory. Thus it is an ideal situation to test if the tachyon field has any cosmological relevance. However, in [7] it was shown that the slow-roll condition for the tachyon inflation is not possible to be achieved in conventional toroidal string compactification and within the validity of small coupling effective theory. To be precise, the slow-roll conditions were not compatible with the string coupling to be much less than one and the dimensionless parameter v, related to the volume of the compact space, to be much greater than one. This leads to the density fluctuations produced during the inflation being incompatible with COBE normalisation. The main source of this criticism stems from the string theory motivated values of the parameters in the tachyon potential i.e. the tension of the brane and the parameters in the gravitational coupling in four dimensions obtained via conventional toroidal string compactification. This objection has cast a shadow on the string motivation of this inflationary scenario. Nevertheless, as shown in [8], if one relaxes the string theory constraints on the above mentioned parameters i.e. if one takes a phenomenological approach, this theory naturally leads to inflation. More interestingly, these authors observed that the tachyon inflation does not lead to the same predictions as standard single field inflation, for example, there is a deviation in one of the second order consistency relations. They also noted that not only the tachyon inflation cannot be ruled out by current observations but also the planned observations probably cannot discriminate between tachyon inflation and single scalar field inflation. This may, however, change in future.

To circumvent the objection in [7] within the conventional string compactification, Piao et al [9] introduced non-minimal coupling of tachyon field to gravity. The idea of non-minimal coupling of scalar fields with gravity has been implemented in the past for various applications [10]. Piao et al claimed that for tachyon cosmology this can predict the observed density perturbation at the cost of the string scale being of the order of a few TeV.

In this note, we point out that the work of Piao et al [9] has a serious algebraic flaw — rectification of which annuls all their predictions. In fact, the correction of this mistake, as we will see in section 2, leads the effective potential being such that the tachyon field cannot roll down and hence the rest of their analysis becomes irrelevant. We reanalyze the inflationary scenario by choosing suitable non-minimal coupling parameters which modify the effective potential as required so that the tachyon field can evolve from its zero value to a non-zero value. Our motive is to see whether the new non-minimal coupling parameters can actually reproduce required values of cosmological observables. But the results are found to be negative, contrary to the claim by the authors of [9]. It is found that the volume parameter v of the compactified space remains to be much less than one if it is chosen to fit the observed spectral index and density perturbations, contrary to the string theory requirement that $v \gg 1$ for the effective field theory of the tachyon-gravity system to be meaningful. We then redo the analysis in a warped compactification background following the analysis of [11] which considered the case of minimal coupling in such a warped background. Our analysis shows that we can have $v \gg 1$ for a wide range of string coupling constant g and the minimum number of D6-branes required to produce the warped background is 10^{13} i.e., marginally less compared to the case in [11].

The paper is organized as follows: in section 2 we discuss the inflationary scenario of non-minimal coupling of tachyon field with gravity correcting the error alluded to earlier. In section 3, we study this problem in a warped background and discuss the possibility of obtaining the cosmological parameters consistent with observations. Section 4 is devoted to discussion of our results.

2. Non-minimally coupled tachyon-gravity

We consider the following action for tachyon non-minimally coupled to gravity [9]

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}Rf(T) - AV(T)\sqrt{1 + Bg^{\mu\nu}\partial_\mu T\partial_\nu T}\right) \tag{2.1}$$

where $f(T)$ is a function of the tachyon T and corresponds to the non-minimal coupling factor. Note that for $f(T) = 1$ this action corresponds to that of the theory with minimal coupling. Here $V(T)$ is the positive definite tachyon potential which has a maximum at $T = 0$ with normalization $V(0) = 1$ and $V \to 0$ as $T \to \infty$. A and B are dimensionful constants which depend on string length and the dilaton which sets the strength of the closed string coupling g. In the conventional compactification (unwarped and constant dilaton) A, B and M_P^2 are given by

$$A = \frac{\sqrt{2}}{(2\pi)^3 g\alpha'^2} \tag{2.2}$$

$$B = 8\ln 2\alpha' \tag{2.3}$$

$$M_P^2 = \frac{v}{g^2\alpha'} \tag{2.4}$$

corresponding to the case of space-filling non-BPS D3-brane in type-IIA theory. Here v is a dimensionless constant related to the volume V_6 of the compactified manifold by

$$v = \frac{2V_6}{(2\pi)^7\alpha'^3}\,. \tag{2.5}$$

For the validity of the effective action we require $g < 1$ and $v \gg 1$. The expressions for A, B and v will change for warped compactification as we will discuss later.

The action (2.1) can be brought to the usual form of Einstein-Hilbert and matter action, for which it is simpler to derive the equation of motion, energy density and pressure, by performing a conformal transformation $g_{\mu\nu}(x) \to f(T)g_{\mu\nu}(x)$, which yields

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}(R - \frac{3}{2}\frac{f'^2}{f^2}g^{\mu\nu}\partial_\mu T\partial_\nu T) - A\tilde{V}(T)\sqrt{1 + Bf(T)g^{\mu\nu}\partial_\mu T\partial_\nu T}\right) \tag{2.6}$$

where $\tilde{V}(T) = V(T)/f^2$ is now the effective potential of the tachyon. The energy density and pressure are found to be

$$\rho = \frac{A\tilde{V}}{\sqrt{1 - Bf\dot{T}^2}} + \frac{3}{4}M_P^2\frac{f'^2}{f^2}\dot{T}^2 \tag{2.7}$$

$$p = -A\tilde{V}\sqrt{1 - Bf\dot{T}^2} + \frac{3}{4}M_P^2\frac{f'^2}{f^2}\dot{T}^2\,. \tag{2.8}$$

The equation of motion of the tachyon and the Friedmann equation are

$$\ddot{T}\left[\frac{1}{1 - Bf\dot{T}^2} + \frac{3}{2}M_P^2\frac{f'^2}{f^2}\frac{\sqrt{1 - Bf\dot{T}^2}}{ABf\tilde{V}}\right] + 3H\dot{T}\left[1 + \frac{3}{2}M_P^2\frac{f'^2}{f^2}\frac{\sqrt{1 - Bf\dot{T}^2}}{ABf\tilde{V}}\right] + \tag{2.9}$$
$$+\dot{T}^2\frac{f'}{2f}\left[\frac{1}{1 - Bf\dot{T}^2} + 3M_P^2\left(\frac{ff'' - f'^2}{f^2}\right)\frac{\sqrt{1 - Bf\dot{T}^2}}{ABf\tilde{V}}\right] + \frac{\tilde{V}'}{Bf\tilde{V}} = 0$$

$$H^2 = \frac{1}{3M_P^2}\frac{A\tilde{V}}{\sqrt{1-Af\dot{T}^2}} + \frac{1}{4}\frac{f'^2}{f^2}\dot{T}^2 \,. \tag{2.10}$$

The inflationary condition is obtained to be

$$Bf\dot{T}^2\left(1+\frac{M_P^2}{ABf\tilde{V}}\frac{f'^2}{f^2}.\sqrt{1-Bf\dot{T}^2}\right) < \frac{2}{3} \tag{2.11}$$

With slow-roll approximation, eqs. (2.9) and (2.10) can be rewritten as

$$3H\dot{T} + \frac{\tilde{V}'}{Bf\tilde{V}} \cong 0 \tag{2.12}$$

$$H^2 \cong \frac{1}{3M_P^2}A\tilde{V} \tag{2.13}$$

where we have assumed that

$$\delta \equiv \frac{3M_P^2}{2}\frac{f'^2}{f^2}\frac{1}{ABf\tilde{V}} \ll 1 \tag{2.14}$$

and which, as we will see, is justified for an appropriate choice of $f(T)$. The slow-roll parameters are found to be

$$\epsilon_1 = \frac{3}{2}Bf\dot{T}^2 = \frac{M_P^2}{2ABf}\frac{\tilde{V}'^2}{\tilde{V}^3} \tag{2.15}$$

$$\epsilon_2 = \frac{2\ddot{T}}{H\dot{T}} = \frac{M_P^2}{ABf}\left(\frac{f'\tilde{V}'}{f\tilde{V}^2} + 3\frac{\tilde{V}'^2}{\tilde{V}^3} - 2\frac{\tilde{V}''}{\tilde{V}^2}\right) \tag{2.16}$$

and

$$\begin{aligned}\epsilon_2\epsilon_3 &= \frac{M_P^2}{(AB)^2}\left(\frac{\dddot{T}}{H\dot{T}} - \frac{\ddot{T}}{\dot{T}}\frac{\dot{H}}{H^2} - \frac{\ddot{T}^2}{H\dot{T}^2}\right) \\ &= \frac{M_P^2}{(AB)^2}\left(\frac{2\tilde{V}'\tilde{V}''}{f^2\tilde{V}^4} - 10\frac{\tilde{V}'^2\tilde{V}''}{f^2\tilde{V}^5} + 9\frac{\tilde{V}'^4}{f^2\tilde{V}^6} - \frac{f''\tilde{V}'^2}{f^3\tilde{V}^3} - \frac{f'\tilde{V}'\tilde{V}''}{f^3\tilde{V}^4} + 5\frac{f'\tilde{V}'^3}{f^3\tilde{V}^5} + 2\frac{f'^2\tilde{V}'^2}{f^4\tilde{V}^4}\right).\end{aligned} \tag{2.17}$$

The usual slow-roll parameters denoted by ϵ and η are related to ϵ_1 and ϵ_2 by $\epsilon = \epsilon_1$ and $\eta = 2\epsilon_1 - \frac{1}{2}\epsilon_2$. Slow-roll conditions are $\epsilon_1 \ll 1$ and $|\epsilon_2| \ll 1$. The end of inflation is given by $|\epsilon_2(T_e)| \simeq 1$, where T_e is the value of the field at the end of inflation.

The number of e-folds between an arbitrary initial value of the field T and T_e is given by

$$N_e(T) \simeq \frac{AB}{M_P^2}\int_{T_e}^{T}\frac{\tilde{V}^2}{\tilde{V}'}dT \,. \tag{2.18}$$

To first order in the slow-roll parameters, the scalar and gravitational power spectra are given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{8\pi^2 M_P^2\epsilon_1} = \frac{1}{24\pi^2}\frac{A}{M_P^4}\frac{\tilde{V}}{\epsilon_1} \tag{2.19}$$

$$\mathcal{P}_g(k) = \frac{2H^2}{\pi^2 M_P^2} \,. \tag{2.20}$$

In the above equations the right hand side is to be evaluated at $aH = k$ where a is the scale factor, H being the Hubble parameter. The tensor-scalar ratio r, the scalar spectral index n and the tensor spectral index n_T are given by:

$$r \equiv \frac{\mathcal{P}_g}{\mathcal{P}_\mathcal{R}} = 16\epsilon_1 \tag{2.21}$$

$$n - 1 \equiv \frac{d \ln \mathcal{P}_\mathcal{R}(k)}{d \ln k} = -2\epsilon_1 - \epsilon_2 = -6\epsilon + 2\eta \tag{2.22}$$

$$n_T \equiv \frac{d \ln \mathcal{P}_g(k)}{d \ln k} = -2\epsilon_1 \,. \tag{2.23}$$

The running of the spectral indices is

$$\frac{dn}{d \ln k} = -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 \tag{2.24}$$

$$\frac{dn_T}{d \ln k} = -2\epsilon_1\epsilon_2 \tag{2.25}$$

The gaussian potential $V(T) = e^{-T^2}$ (motivated from boundary string field theory), gives a good description for small T and can be assumed to be accurate for the inflationary epoch. With this the effective potential becomes $\tilde{V} = e^{-T^2}/f^2$. In general $f(T)$ can be expanded as $f(T) = 1 + \sum_{i=1} c_i T^{2i}$. Expanding $\tilde{V}$ in powers of T we get

$$\tilde{V}(T) = 1 - (1 + 2c_1)T^2 + (\frac{1}{2} + 2c_1 - 2c_2 + 3c_1^2)T^4 + \cdots \,. \tag{2.26}$$

With this effective potential, the requirement that in the neighbourhood of $T = 0$ we should have the slow-roll condition $|\eta| \ll 1$ gives the constraint:

$$\frac{4M_P^2}{AB}(1 + 2c_1) \ll 1 \tag{2.27}$$

Note that for $c_1 = 0$ this gives the unjustifiable slow-roll constraint $g/v \gg 127$ [7] in the minimal coupling case. However, the choice $c_1 = -1/2$ makes the slow-roll condition eq. (27) automatically satisfied, as noted in [9]. Further, if all other c_i's are chosen to be zero, the expansion of $\tilde{V}$ becomes

$$\tilde{V}(T) = 1 + \frac{1}{4}T^4 + \cdots \,. \tag{2.28}$$

This potential has a minimum at $T = 0$ and it increases as T increases. Thus, it is not suitable to have a slow-roll inflationary scenario with this potential, where the inflaton field rolls down the slope of the potential from a lower value to higher value.

Piao et al have made the following error. They had the following expansion of $\tilde{V}$

$$\tilde{V}(T) = 1 - (1 + 2c_1)T^2 + \left(\frac{1}{2} + 2c_1 - 2c_2 - c_1^2\right)T^4 + \cdots \tag{2.29}$$

with the incorrect term $-c_1^2$ instead of $3c_1^2$ in the coeffiecient of T^4, as in eqn.(26). With the choice $c_1 = -1/2$ and all other c_i's put equal to zero, the above expression then is

$$\tilde{V}(T) = 1 - \frac{3}{4}T^4 + \cdots$$

with the potential having a maximum at $T = 0$. Thus all their remaining analysis based on this incorrect potential breaks down. Further, they claimed that comparing with observational data on $\mathcal{P}_R$ and $\mathcal{P}_g$ it is possible to achieve $g < 1$ and $v \gg 1$. However, when observational data on spectral index etc are taken into account this claim is also found to be incorrect.

It is, nevertheless, possible to have a feasible model of slow-roll inflation, with the potential having a maximum at $T = 0$, by invoking non-minimal coupling of tachyon with gravity with the choice $c_1 = -1/2$ (so that T^2 term vanishes) and keep at least upto T^4 term in $f(T)$, choosing c_2 in such a way that the coefficient of T^4 in the expansion of $\tilde{V}$ is negative. We choose, accordingly, c_2 to be $1/4$ and all other c_i's to be zero. In this case we have

$$\tilde{V}(T) = 1 - \frac{1}{4}T^4 + \cdots .$$

This effective potential has maximum at $T = 0$ as required. For our above choice of c_1 and c_2, we find that δ is of the same order as ϵ_2 (or η). Thus the requirement of $\delta \ll 1$ is automatically satisfied within the slow-roll regime and is not an independent requirement.

Following [8] we find that $\tilde{V}'' < \tilde{V}'^2/\tilde{V}$ for all values of T. Inflation can take place in either region I, given by $\tilde{V}'' \leq 0$, or region II, given by $0 \leq \tilde{V}'' \leq \tilde{V}'^2/\tilde{V}$. Here region I ends roughly at $T = 1.12$. This means that T_e can at most have this value and correspondingly T_*, which is the value of the tachyon field at roughly 60 e-folds when the cosmological scale crosses the horizon, is bounded to a maximum value of roughly 0.5. We restrict ourselves to region I. All observables, namely, n, $\mathcal{P}_R$, r and $dn/d\ln k$ are to be evaluated at T_*.

Clearly the advantage of introducing non-minimal coupling with the choice of parameters $c_1 = -1/2$ and $c_2 = 1/4$ is to remove the slow-roll constraint that $AB/4M_P^2 \gg 1$ and hence it is possible to choose it to be small. It is not arbitrary, however, and must be chosen to fit observations. We outline below the strategy of our numerical estimation:

1. From the condition for the end of inflation, $\epsilon_2(T_e) = 1$, T_e is obtained as a function of AB/M_P^2.

2. Fixing N_e to be 60 we evaluate T_* as a function of T_e and consequently a function of AB/M_P^2.

3. Then we find the range of AB/M_P^2 for which the spectral index is in the range $0.94 \leq n \leq 1$.

4. $\mathcal{P}_R$ independently depends on A/M_P^4 and its range cannot be fixed only from AB/M_P^2. We use the observational input $\mathcal{P}_R \leq 0.71 \times 2.9 \times 10^{-9}$ [12] in eqn.(19) to get the range for A/M_P^4.

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AB/M_P^2	A/M_P^4	g	v
60	9.74×10^{-11}	4.73×10^{-15}	2.48×10^{-18}
1	1.08×10^{-13}	1.89×10^{-14}	5.94×10^{-16}
0.01	1.21×10^{-17}	2.12×10^{-14}	6.68×10^{-14}
10^{-4}	1.22×10^{-21}	2.13×10^{-14}	6.71×10^{-12}
10^{-6}	1.22×10^{-25}	2.13×10^{-14}	6.71×10^{-10}
10^{-8}	1.22×10^{-29}	2.13×10^{-14}	6.71×10^{-8}
10^{-10}	1.22×10^{-33}	2.13×10^{-14}	6.71×10^{-6}
10^{-13}	1.7×10^{-39}	2.96×10^{-14}	1.93×10^{-3}

Table 1: Values of g and v calculated from observed values of A and B.

We quote below the range of values for AB/M_P^2, A/M_P^4 and the observables r and $dn/d\ln k$ obtained from our numerical estimates. The upper bound for AB/M_P^2 is obtained from the maximum value of T_e in region I. This constrains AB/M_P^2 as

$$10^{-13} \leq \frac{AB}{M_P^2} \leq 69\,. \tag{2.30}$$

The range for A/M_P^4 is:

$$1.7 \times 10^{-39} < \frac{A}{M_P^4} < 9.74 \times 10^{-11}\,. \tag{2.31}$$

The range for r is obtained to be

$$5.6 \times 10^{-32} < r < 2.5 \times 10^{-3}\,.$$

The running of the spectral index is negative and is found to lie in the range

$$-8.9 \times 10^{-4} < \frac{dn}{d\ln k} < -5.09 \times 10^{-4}\,.$$

r and $dn/d\ln k$ are well within the experimental bounds.

An analysis of the above estimates in terms of the string theory parameters g and v is in order. For each corresponding set of values of AB/M_P^2 and A/M_P^4 we can solve for g and v as

$$g = \frac{(A/M_P^4)}{(AB/M_P^2)^2} \times 0.175 \tag{2.32}$$

$$v = \frac{(A/M_P^4)}{(AB/M_P^2)^3} \times 0.0055 \tag{2.33}$$

We tabulate the corresponding values in table 1 for values of n decreasing from 0.963 to 0.94.

It is clear from the above table that, though g is always much less than one, it is impossible to constrain v to be much greater than one and still satisfy observation. Thus we conclude that even if we invoke non-minimal coupling with fine tuned coupling parameters c_1 and c_2, we cannot achieve $g < 1$ and $v \gg 1$, satisfying the observational constraints on cosmological parameters.

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3. Tachyon inflation in warped background

Since within conventional compactification it is not possible to obtain physical parameters of inflation consistent with observation, keeping $g < 1$ and $v \gg 1$ even with non-minimal coupling, it becomes important to look for alternative string compactifications with a hope that this may improve the values of the parameters and allow us to have $g < 1$ and $v \gg 1$, thus freeing tachyon inflation from the criticism mentioned earlier. Already some progress has been achieved in this direction, recently, by Raeymakers [11]. The author considered a warped compactification, where the four dimensional metric contains an overall factor which can vary over the compactified space [13]. The dilaton was allowed to vary over the compact manifold as well. The latter has two advantages. Since the parameters governing the tachyon action depends upon dilaton, one has more freedom by allowing it to vary. Secondly, the varying dilaton contributes to the warp factor in the Einstein frame. Such a warped background could be obtained by wrapping a large number of space-filling D-branes on a cycle of the compact manifold since the back reaction produces a "throat region" with enough warping and varying dilaton. It is found that the parameter range required for inflation can be accommodated in the background of D6-branes wrapping a three-cycle in type-IIA theory. The important ingredient which goes in here to make the inflation possible is the fact that the string coupling decreases faster than the (string frame) warp factor as one approaches the branes.

The effective field theory that describes inflation when the tachyon is coupled minimally to gravity [6] is the same as eqn.(1) when we set $f(T) = 1$, i.e.,

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - AV(T)\sqrt{1 + Bg^{\mu\nu}\partial_\mu T\partial_\nu T}\right). \tag{3.1}$$

Warping is introduced by considering the ten dimensional string frame metric of the form

$$ds^2 = e^{2C(y)}g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n \tag{3.2}$$

where e^{2C} is the warp factor and can take very small values. Moreover, if the dilaton is allowed to vary over the compact manifold as

$$\phi = \phi_0 + \phi(y)\,, \tag{3.3}$$

one finds that the four dimentional Planck mass in such warped compatification is

$$M_P^2 = \frac{\tilde{v}}{g^2\alpha'}\,. \tag{3.4}$$

Here $g = e^{\phi_0}$ and the 'warped volume' $\tilde{v}$ is

$$\tilde{v} = \frac{2}{(2\pi)^7\alpha'^3}\int d^6y\sqrt{g_6}e^{-2\phi+2C}\,. \tag{3.5}$$

$\tilde{v}$ can be taken to be the same order as v by choosing the average value of $e^{-2\phi+2C}$ to be of order one as taken in [11]. However, locally the y-dependent functions affect A and B in eqn. (3.1). For example, when the non-BPS D3-brane is embedded in such a background, it can be seen that A and B become

$$A = \frac{\sqrt{2}e^{4C-\phi}}{(2\pi)^3 g\alpha'^2} \tag{3.6}$$

$$B = 8\ln 2\alpha' e^{-2C}\,. \tag{3.7}$$

Note that the functions C and ϕ are not arbitrary but are subject to the solutions of equations of motion derived from supergravity theory. The warping modifies the slow-roll constraint $AB/M_P^2 \gg 1$ as

$$\frac{g}{v}e^{2C-\phi} \gg 127\,. \tag{3.8}$$

which can be satisfied in this background where locally we have

$$e^{2C-\phi} \gg 1\,. \tag{3.9}$$

This condition can be achieved by wrapping a large number N of D6-branes over a three cycle.

Within the regime of supergravity approximation one can derive the following expressions for the warp factor and the dilaton:

$$e^{2C} = (gN_{\rm min})^{-2/3} \tag{3.10}$$

$$e^{-\phi} = gN_{\rm min} \tag{3.11}$$

where $N_{\rm min}$ is the minimum number of branes. In this case the condition (3.8) becomes

$$\frac{g^{4/3}}{v} \gg \frac{127}{N_{\rm min}^{1/3}}\,.$$

Thus for satisfying the slow-roll condition, $N_{\rm min}$ is required to be at least of the order of 10^6 [11]. Further, the author found that in order to satisfy the observational constraints on density perturbation and spectral index, the minimum number of branes required is of the order of 10^{14}.

To highlight the allowed values of g and v we give below their relationship, along with the warp factors, with the parametric values of A and B obtained from observational data:

$$ge^{\phi} = \frac{(A/M_P^4)}{(AB/M_P^2)^2} \times 0.175 \tag{3.12}$$

$$ve^{-2C+2\phi} = \frac{(A/M_P^4)}{(AB/M_P^2)^3} \times 0.0055\,. \tag{3.13}$$

Unlike the unwarped case we now have an extra parameter due to warping, which can be used to obtain the required values of g and v. Eliminating e^{ϕ} and e^{2C} using eqs. (3.10) and (3.11) in the above equations we get a relation between v and g:

$$v = g^{4/3}y^{-1/3}x^{-1/3} \times 0.057 \tag{3.14}$$

AB/M_P^2	A/M_P^4	n	ge^{ϕ}	$ve^{-2C+2\phi}$	g
70.9	3.02×10^{-10}	0.94	1.05×10^{-14}	4.66×10^{-18}	$g > 0.10$
100	6.54×10^{-10}	1.0×0.953	10^{-14}	10^{-18}	$g > 0.14$
500	3.69×10^{-9}	0.969	2.58×10^{-15}	8.12×10^{-19}	$g > 0.32$
900	5.92×10^{-9}	0.9696	1.28×10^{-15}	4.47×10^{-20}	$g > 0.41$
1100	6.93×10^{-9}	0.9697	1.0×10^{-16}	2.86×10^{-20}	$g > 0.45$

Table 2: Range of g estimated for $v > 1$, for each pair of observed values of A and B for minimal coupling.

AB/M_P^2	A/M_P^4	ge^{ϕ}	$ve^{-2C+2\phi}$	g	v for $g = 0.1$
60	9.74×10^{-11}	4.73×10^{-15}	2.48×10^{-18}	$g > 0.07$	1.47
1	1.08×10^{-13}	1.89×10^{-14}	5.94×10^{-16}	$g > 0.0049$	55.56
0.01	1.21×10^{-17}	2.12×10^{-14}	6.68×10^{-14}	$g > 1.61 \times 10^{-4}$	5.35×10^{3}
10^{-4}	1.22×10^{-21}	2.12×10^{-14}	6.71×10^{-12}	$g > 5.09 \times 10^{-6}$	5.33×10^{5}
10^{-6}	1.22×10^{-25}	2.12×10^{-14}	6.71×10^{-10}	$g > 1.61 \times 10^{-8}$	5.33×10^{7}
10^{-8}	1.22×10^{-29}	2.13×10^{-14}	6.71×10^{-8}	$g > 5.09 \times 10^{-9}$	5.33×10^{9}
10^{-10}	1.22×10^{-33}	2.13×10^{-14}	6.71×10^{-6}	$g > 1.61 \times 10^{-10}$	5.33×10^{11}
10^{-13}	1.7×10^{-39}	2.98×10^{-14}	1.93×10^{-3}	$g > 9.85 \times 10^{-13}$	4.78×10^{14}

Table 3: Range of g estimated for $v > 1$, for each pair of observed values of A and B for non-minimal coupling. The last column gives v for a fixed value of g.

where x denotes AB/M_P^2 and y denotes A/M_P^4. To have $v \gg 1$ the condition on g is

$$g > y^{1/4} x^{1/4} \times 8.62\,. \tag{3.15}$$

The range of values obtained are tabulated in table 2. Each row of table 2 corresponds to a different value of n, which increases from 0.94 to 0.97. The sixth column gives the constraint on g when we demand $v \gg 1$.

We see from the table 2 that it is possible to have $v \gg 1$, but only for $g > 0.1$.

We now repeat this analysis, i.e., to include warping for the non-minimal coupling case discussed in section 2. This is in expectation that since the non-minimal coupling removes the slow-roll constraint and warping improves the value of v, the parameters and the minimum number of branes can be improved.

The numerical values we get are tabulated below. This is essentially the same as table 1, but with the values of g and v reinterpreted in the light of the warped background, as presented in table 2.

The last column of table 3 quotes the values of v for $g = 0.1$ to highlight the improved values of v. Thus, $v \gg 1$ is achieved over a wide range of values of g. The minimum number of branes required in this case turns out to be of the order of 10^{13}, which is a marginal decrease of one order of magnitude from the minimal case.

We can also estimate, for this model, the scale of inflation which is found to range from 10^{16} to 10^{9} GeV. This is in contrast to the range from 9.8×10^{16} to 1.7×10^{16} GeV in the case of minimal coupling in a warped background [11].

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4. Discussion

In this paper, we have shown that the non-minimal coupling of the form $-RT^2/2$ between tachyon field and gravity, proposed in [9], is not consistent with slow-roll inflation in cosmology. We correct this problem by adding another non-minimal coupling of the form $RT^4/4$. However, we found that even if slow-roll conditions are satistied, it does not meet the fundamental requirement that one needs $g < 1$ and $v \gg 1$ for the validity of low energy effective theory. Thus, non-minimal coupling alone, as analyzed in section 2, does not circumvent the problems in tachyon cosmology. We then invoke the warped compactification with a varying dilaton field in the compact space and analyze this problem, closely following [11]. We find that this model can be a viable model for inflation in cosmology. However, we still need a large number of D6-branes (10^{13}, compared to 10^{14} in the minimal case) for producing the required warped background. We note that these non-minimal coupling terms in the effective theory can arise by computing the S-matrix elements of open string tachyons and closed string gravitons [14]. Though we kept only quadratic and quartic terms in tachyon field, in principle we should consider the coupling of tachyons to all orders to obtain an exact form involving non-minimal couplings. We will return to this issue in future. It is possible that the exact non-minimal coupling to all orders in tachyon field can produce an effective potential which can have a local minimum. Such a minimum can help in addressing the reheating issue, just as the case of any other scalar field driven cosmology having a potential with a true minimum. This possibly will be a suitable alternative to the proposal of [16] in the context of tachyon inflation. We also point out that our work assumes the possibility of stabilizing various scalar moduli fields and other requirements of warped compactification [15] just as in [11].

Acknowledgments

We thank Ashoke Sen and Sandip Trivedi for many useful discussions.

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